# 3. EXPERIMENT #1A:

## Non-harmonic Excitation Of Second Order Systems

This experiment includes two parts. The first part tests the characteristic responses of several system types to a step input including position, velocity, and acceleration at $t = 0^+$ and at steady state. The second part shows the important properties of proportionality and superposition.

### *3.1 Step Response of Various System Types*

This experiment characterizes the response of various system *types* shown in Figure 3.1 to a torque step input. As will be studied here and later in the context of harmonically driven response, these systems have qualitatively different dynamic behavior. Hence the term "types" has a special meaning in the parlance of dynamics. Here, "a", "b", and "c" in Figure 3.1 are of system types 2, 1, and 0 respectively.

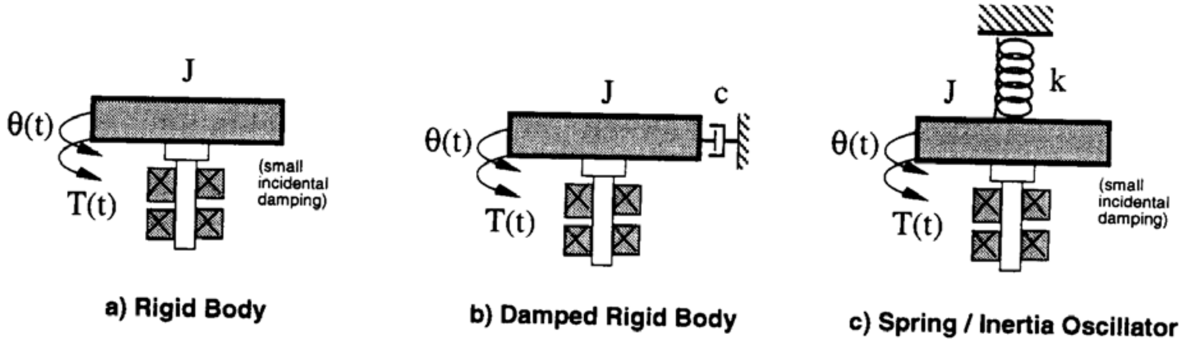

Figure 3.1 System types for step and impulse tests.

Before beginning the step response experiments, a brief demonstration of electromechanically actuated spring and damping ($k$ and $c$) effects is given. This implementation is used to provide damping in the present experiment and both damping and spring action later. Electromechanical actuation allows for more precise control of these parameters. The implementation of these dynamic effects via control action also forms the building blocks of motion control systems theory and practice which may be studied in later courses.

Procedure (Electromechanical spring & damper)

1. Remove the upper and middle disks from the mechanism and secure 4 brass weights at R = 9.0 cm on the lower disk. Thus, set up the mechanism in Configuration 7 (Table 2.1). After safety checking the system (per section 1.2) spin the inertia by hand to verify that it rotates freely and to get a feel for its dynamic response to the torque that you apply.
2. Go to **Setup Force + Spring + Damper** driving function (**Setup** menu) and set the spring constant to $\boldsymbol{k} = 0.1$ N-m/rad ($\boldsymbol{c} = 0$). Select **OK** to return to the **Setup Driving Function** box.

   In this and all future work, be sure to stay clear of the mechanism before doing the next step. Selecting **Enable Driving Function** immediately implements the specified motor control action; if improper driving function coefficients have been entered or an anomalous data states exist in the controller, the motor and mechanism may react violently. If the system appears stable after enabling the driving function, spin the disk with a light, non sharp object (e.g. a plastic ruler) to verify stability prior to touching system – see Section 1.2.

Select **Enable Driving Function**, then OK. Verify system stability as per the above notation. Now manually move the lower disk back and forth slowly. The motor drive has been programmed to supply a torque proportional to and in the opposite direction of the position of the disk, i.e. you are feeling an equivalent proportional spring. Can you feel the opposing torque increase as the position from center increases? Note: Use light fingertip torque only. Do not apply excessive torque to the disk as this may result in overheating of the drive electronics. Do not apply force for longer than 5 seconds continuously and wait approximately 20 seconds before reapplying torque.

3. Repeat Step 2 at various spring constant levels up to but not exceeding 0.2 N-m/rad (e.g. ***k*** = 0.025, 0.05 N-m/rad, etc.). You may wish to release the disk at some nonzero position and note the general effect of changing spring constant on frequency of oscillation and damping.
4. Return to **Setup Force + Spring + Damper** driving function (**Setup** menu) and set the damping constant to ***c*** = 0.01 N-m/(rad/s) (***k*** = 0). Select **OK** to return to the Setup Driving Function box. Select **Enable Driving Function**, then **OK**. Again verify stability as per the above notation.

   Manually move the disk back and forth. This time, the motor drive has been programmed to supply a torque proportional to and in the opposite direction of the angular velocity of the disk, i.e. you are feeling viscous friction. Can you feel the opposing torque increase as speed increases? Note: Use light fingertip torque only. Again, do not apply torque for longer than 5 seconds continuously and wait approximately 20 seconds before reapplying torque.
5. Repeat Step 4 at various levels of damping from 0 up to but not exceeding 0.05 N-m/(rad/s) (e.g. ***c*** = 0.005, 0.02 N-m/(rad/s), etc.).

Procedure (Step response)

6. Set up the mechanism in Configuration 7 (Table 2.1). After safety checking the system (per Section 1.2) spin the inertia by hand to verify that it rotates freely.
7. "Enable" the **Force (Torque)** driving function via the **Setup Driving Function** dialog box. Setup a unidirectional **Step** input shape of 250 mN-m amplitude, 200 ms duration, and 1 repetition via the **Input Shape** dialog box with the **Unidirectional** box checked. Setup to acquire Encoder 1 Position data once every 2 servo cycle.
8. Execute the step maneuver and plot the resulting position, velocity, and acceleration data over the 200 ms. interval. To obtain proper scaling, plot position on the left axis, and velocity and acceleration on the right axis. You may wish to plot one variable on a separate graph for better resolution. Save your plot data.
9. Apply damping to the system via **Setup Force + Spring + Damper** using ***c*** = 0.02 N-m/(rad/s) and ***k*** = 0. **Enable** the driving function. Again after safety checking the controller, rotate the inertia by hand to verify free/damped rotation. Reset the encoders to zero via the **Utility** menu. Do not apply excessive torque or torque for longer than 5 seconds.

10. Setup a unidirectional **Step** input per Step 7 but with the duration changed to 1000 ms. Execute the same step maneuver as in Step 8 and plot the resulting position, velocity, and acceleration data over the 1.0 s. interval. Save your plot data.
11. Clamp the torsion shaft at the upper disk location (Configuration 5, Table 2.1) and verify that the disk rotates freely within the constraint of the torsional spring shaft. Do not apply excessive torque (no more than 20 degrees disk rotation) to the shaft.
12. "Enable" the **Force (Torque)** driving function per step 7 above but with the step amplitude and duration changed to 20 mN-m and 2000 ms respectively. Obtain the system step response per step 8 over the 2.0 s interval.

**Exercises:**

A. What are the salient position, velocity, and acceleration response characteristics of the three system types shown in Figure 3.1 to a step input? You may neglect the effects of mechanism friction in your descriptions. Use Table 3.1 to record your observations – you may photocopy or otherwise reproduce the table. Note the initial position, velocities, and accelerations (at $t = 0^+$) in each case. Describe the relative phasing of the position, velocity, and acceleration curves in the spring/inertia system response. What are the relative amplitudes of oscillation between the position, velocity, and acceleration curves?

B. Explain your results in terms of the particular solutions to Eq. (2.1) for each system type. You may assume $T(t) = \alpha u(t)$ where $u(t)$ is the unit step function, and $\alpha$ is the force magnitude scaling. What are the ideal initial positions, velocities, and accelerations (immediately following the step) in each case? What are the steady state positions? Again, you may neglect unmodeled mechanism friction in your analysis.

C. Explain any significant differences from theoretically ideal results.

Table 3.1 Observations from step response tests.

| **System Type** | **Position Shape** $\theta(t)$ | **Velocity Shape** $\dot{\theta}(t)$ | **Acceleration Shape** $\ddot{\theta}(t)$ |
|---|---|---|---|
| Rigid Boday | | | |
| Damped Rigid Body | | | |
| Spring / Inertia Oscillator | | | |

### *3.2 Linearity Principles: Proportionality & Superposition*

Two important properties of linear systems are proportionality and superposition. For convenience of review, these principles are depicted in Figure 3.2. In this experiment, these concepts are demonstrated on the spring / inertia oscillator shown in Figure 3.3. Here we use impulsive forcing functions because the results lead to a conceptual and analytical approach for obtaining the response to an arbitrary forcing function as studied in the next section. The concepts demonstrated here however, hold for general continuous functions as well.

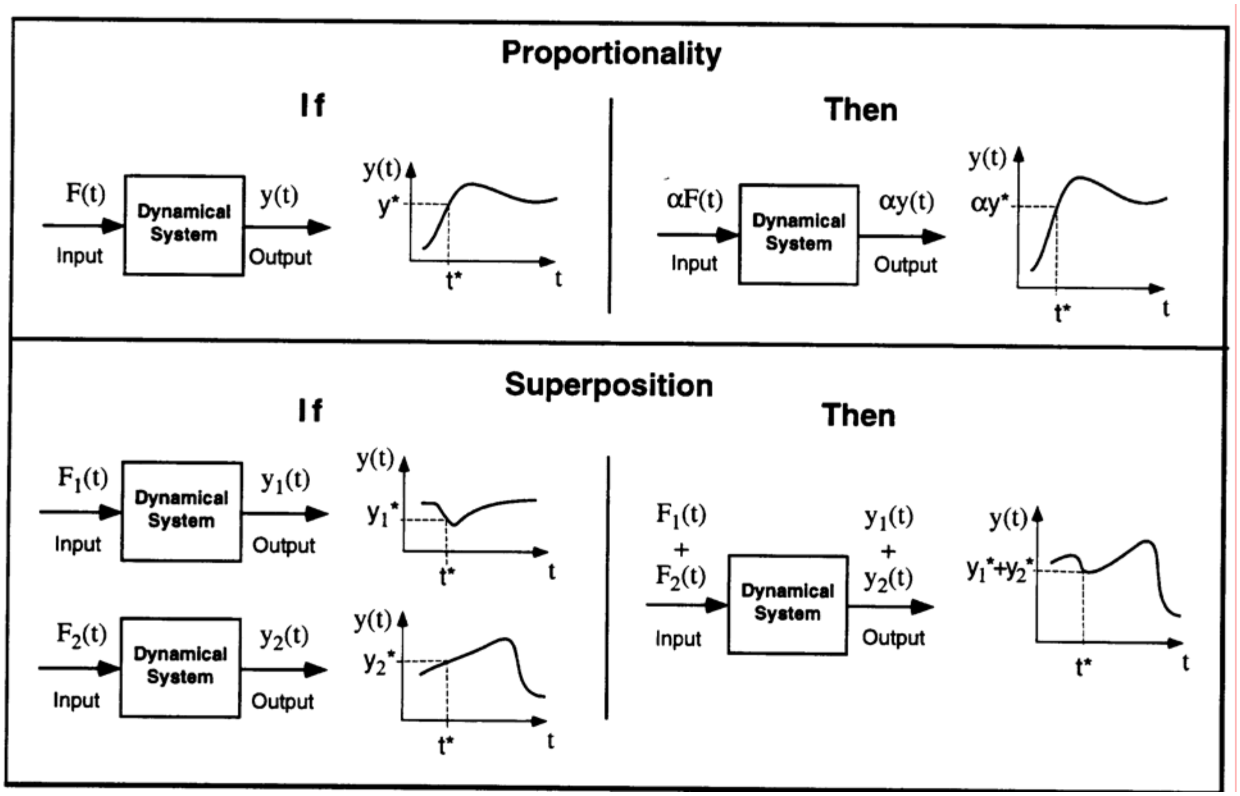

Figure 3.2 The principles of proportionality and superposition.

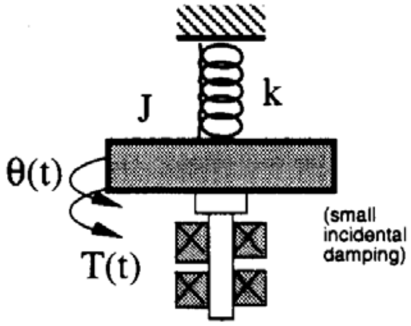

Figure 3.3 System for tests of linearity principles.

Procedure (Proportionality):

1. Set up the mechanism in Configuration 5 (Table 2.1). After safety checking the system (per Section 1.2) rotate the inertia by hand to verify that it rotates freely within the constraint of the torsional spring shaft. Do not apply excessive torque (no more than 20 degrees disk rotation) to the shaft.
2. "Enable" the **Force (Torque)** driving function via the **Setup Driving Function** dialog box. Setup a unidirectional **Impulse** input shape of 50 mN-m, 50 ms pulse width, 1 repetition, and 1950 ms dwell time. Setup to acquire Encoder 1 Position and Drive Input data once every 1 servo cycles.
3. Execute the impulse maneuver (select **Extended Data Sampling** in the **Execute** box) and plot the resulting Encoder 1 position, and Drive Input data over a 2.0s. interval. To obtain proper scaling, it is best to plot position on the left axis, and drive input on the right axis. Print your plot.
4. Change the impulse amplitude to 100 mN-m and repeat Step 3 above.

**Exercise:**

A. How does changing the forcing function input amplitude affect the output (neglecting mechanism friction effects)? Explain using the terminology defined in Figure 3.2.

Procedure (Superposition):

5. Change the impulse width to 25 ms (2000 mN-m amplitude) and repeat Step 3 above. Measure the times when the output oscillation phase is at $\pi$, $3\pi/2$, and $2\pi$ of the sine function (i.e. first negative slope zero crossing, first minimum, and first positive slope zero crossing) and identify them as $t_1$, $t_2$, and $t_3$ respectively. You should "zoom" your plot scaling to obtain best resolution to measure these times.
6. Change the number of impulse repetition to 2 and set the dwell period such that the second pulse is centered about $t_1$. (You should properly account for the initial and second pulse durations.) Setup to acquire Encoder 1 Position and Drive Input data once every 1 servo cycle. Execute the double pulse input (select **Extended Data Sampling** in the **Execute** box) and plot the position and input data over a 2 s. interval. Print the plot.
7. Repeat Step 6 with the second pulse centered about $t_2$. Repeat again for $t_3$.

**Exercise:**

A. Explain the system output for each of the three cases in terms of the principle of superposition using the terminology defined in Figure 3.2. You may consider $F_1(t)$ to be the initial impulse, and $F_2(t)$ to be the second. Select data points at 3 distinct times from one of the three double pulse cases (your choice, but note that using data points fairly early in the response usually results in measurements that are more consistent with theory because friction effects are less dominant.). At least two of these times should be greater than the second pulse delay time, $t_i$ ($i$ = 1,2,or 3) for whichever case you choose. Show based on measurements of the initial single pulse response (Step 5) how the values for your double pulse case are predicted by with superposition.