

### 3. EXPERIMENT #1B:

#### Non-harmonic Excitation Of Second Order Systems

This experiment includes two parts. The first part tests the characteristic responses of several system types to a step input including position, velocity, and acceleration at  $t = 0^+$  and at steady state. The second part shows the important properties of proportionality and superposition.

##### 3.1 Step Response of Various System Types

This experiment characterizes the response of various system *types* shown in Figure 3.1 to a force step input. As will be studied here and later in the context of harmonically driven response, these systems have qualitatively different dynamic behavior. Hence the term “types” has a special meaning in the parlance of dynamics. Here, “a”, “b”, and “c” in Figure 3.1 are of system types 2, 1, and 0 respectively.

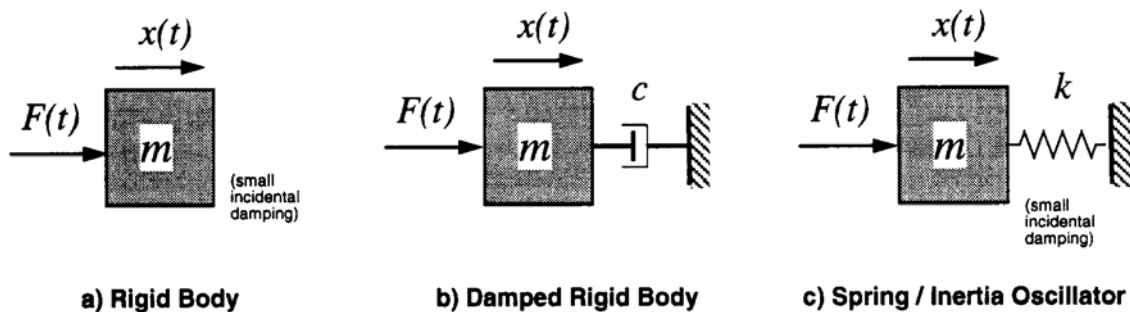


Figure 3.1 System types for step impulse response tests.

Before beginning the step response experiments, a brief demonstration of electromechanically actuated spring and damping ( $k$  and  $c$ ) effects is given. This implementation is used to provide damping in the present experiment and both damping and spring action later. Electromechanical actuation allows for more precise control of these parameters. The implementation of these dynamic effects via control action also forms the building blocks of motion control systems theory and practice which may be studied in later courses.

##### Procedure (Electromechanical spring & damper)

1. Remove the clamps from the first mass carriage and clamp the second carriage in its mid-travel position. Attach 4 brass weights to the first mass carriage. The mechanism should now be set up in Configuration 7 (Table 2.1). Move the carriage back and forth by hand to verify that it moves freely and to get a feel for dynamic response of the inertia to the applied force.
2. Go to **Setup Force + Spring + Damper** driving function (**Setup** menu) and set the spring constant to  $k = 250$  N/m ( $c = 0$ ). Select **OK** to return to the **Setup Driving Function** box.

In this and all future work, be sure to stay clear of the mechanism before doing the next step. Selecting **Enable Driving Function** immediately implements the specified motor control action; if improper driving function coefficients have been entered or an anomalous data

states exist in the controller, the motor and mechanism may react violently. If the system appears stable after enabling the driving function, displace the carriage with a light, non sharp object (e.g. a plastic ruler) to verify stability prior to touching system – see Section 1.2.

Select **Enable Driving Function**, then OK. Verify system stability as per the above notation. Now manually move the first carriage back and forth slowly. The motor drive has been programmed to supply a force proportional to and in the opposite direction of the position of the carriage, i.e. you are feeling an equivalent proportional spring. Can you feel the opposing force increase as the position from center increases? Note: Use light fingertip force only. Do not apply excessive force to the carriage as this may result in overheating of the drive electronics. Do not apply force for longer than 5 seconds continuously and wait approximately 20 seconds before reapplying force.

3. Repeat Step 2 at various spring constant levels of 100 up to but not exceeding 500 N/m. You may wish to release the carriage at some nonzero displacement (avoid contacting the limit switch) and note the general effect of changing spring constant on frequency of oscillation and damping.
4. Return to **Setup Force + Spring + Damper** driving function (**Setup** menu) and set the damping constant to  $c = 50$  N/(m/s) ( $k = 0$ ). Select **OK** to return to the Setup Driving Function box. Select **Enable Driving Function**, then **OK**. Again verify stability as per the above notation.

Manually move the carriage back and forth. This time, the motor drive has been programmed to supply a force proportional to and in the opposite direction of the velocity of the carriage, i.e. you are feeling viscous friction. Can you feel the opposing force increase as speed increases? Note: Use light fingertip force only. Again, do not apply force for longer than 5 seconds continuously and wait approximately 20 seconds before reapplying force.

5. Repeat Step 4 at various levels of damping from 0 up to but not exceeding 250 N/(m/s).

#### Procedure (Step response)

6. The mechanism should now be set up in Configuration 7 (Table 2.1). Set  $k = 0$ ,  $c = 0$  via the **Setup Force + Spring + Damper** driving function (**Setup** menu). After safety checking the system (per Section 1.2) move the first carriage by hand to verify that it slides freely.
7. Set the first mass carriage near its left limit of travel (toward the motor). “Enable” the **Force (Torque)** driving function via the **Setup Driving Function** dialog box. Setup a unidirectional **Step** input shape of 5 N amplitude, 120 ms duration, and 1 repetition via the **Input Shape** dialog box. Make certain that the **Unidirectional** box is checked. Setup to acquire Encoder 1 Position data once every 1 servo cycle.
8. Execute the step maneuver. Note that the carriage will contact the limit switch so you must “re-implement” the driving function prior to executing the next maneuver. Plot the resulting position, velocity, and acceleration data over the first 100 ms interval. To obtain proper scaling, plot position on the left axis, and velocity and acceleration on the right axis. You may wish to plot one variable on a separate graph for better resolution. You should adjust the scaling of each axis to fully magnify the data. Save you plot data.

9. Apply damping to the system via **Setup Force + Spring + Damper** using  $c = 100 \text{ N/(m/s)}$  and  $k = 0$ . Again after safety checking the controller, slide the carriage by hand to verify free/damped motion. Do not apply excessive force or force for longer than 5 seconds. Reset the encoders to zero via the **Utility** menu.
10. Increase the step duration to 500 ms with all other parameters unchanged and execute the maneuver. Plot the resulting position, velocity, and acceleration data over the first 400 ms. Save your plot data.
11. Install the medium stiffness spring on the first carriage at Location #1 (Configuration 8, Table 2.1) and verify that the carriage slides freely within the constraint of the spring.
12. “Enable” the **Force (Torque)** driving function. Increase the step duration to 3000 ms and execute the maneuver. Plot the resulting position, velocity, and acceleration data over the first 2500 ms. Save your plot data.

### Exercises:

- A. What are the salient position, velocity, and acceleration response characteristics of the three system types shown in Figure 3.1 to a step input? You may neglect the effects of mechanism friction and motor/amplifier nonlinearities in your descriptions. Use Table 3.1 to record your observations – you may photocopy or otherwise reproduce the table. Note the initial position, velocities, and accelerations (at  $t = 0^+$ ) in each case. Describe the relative phasing of the position, velocity, and acceleration curves in the spring/inertia system response. What are the relative amplitudes of oscillation between the position, velocity, and acceleration curves?
- B. Explain your results in terms of the particular solutions to Eq. (2.1) for each system type. You may assume  $F(t) = \alpha u(t)$  where  $u(t)$  is the unit step function, and  $\alpha$  is the force magnitude scaling. What are the ideal initial positions, velocities, and accelerations (immediately following the step) in each case? What are the steady state positions? Again, you may neglect unmodeled mechanism friction in your analysis.
- C. Explain any significant differences from theoretically ideal results.

### 3.2 Linearity Principles: Proportionality & Superposition

Two important properties of linear systems are proportionality and superposition. For convenience of review, these principles are depicted in Figure 3.2. In this experiment, these concepts are demonstrated on the spring / inertia oscillator shown in Figure 3.3. Here we use impulsive forcing functions because the results lead to a conceptual and analytical approach for obtaining the response to an arbitrary forcing function as studied in the next section. The concepts demonstrated here however, hold for general continuous functions as well.

Table 3.1 Observations from step response tests.

System Type	Position Shape $x(t)$	Velocity Shape $\dot{x}(t)$	Acceleration Shape $\ddot{x}(t)$
Rigid Body			
Damped Rigid Body			
Spring / Inertia Oscillator			

Procedure (Proportionality):

1. Set up the mechanism in Configuration 8 (Table 2.1). After safety checking the system (per Section 1.2) slide the first carriage by hand (avoid contacting the limit switches) to verify that it moves freely.

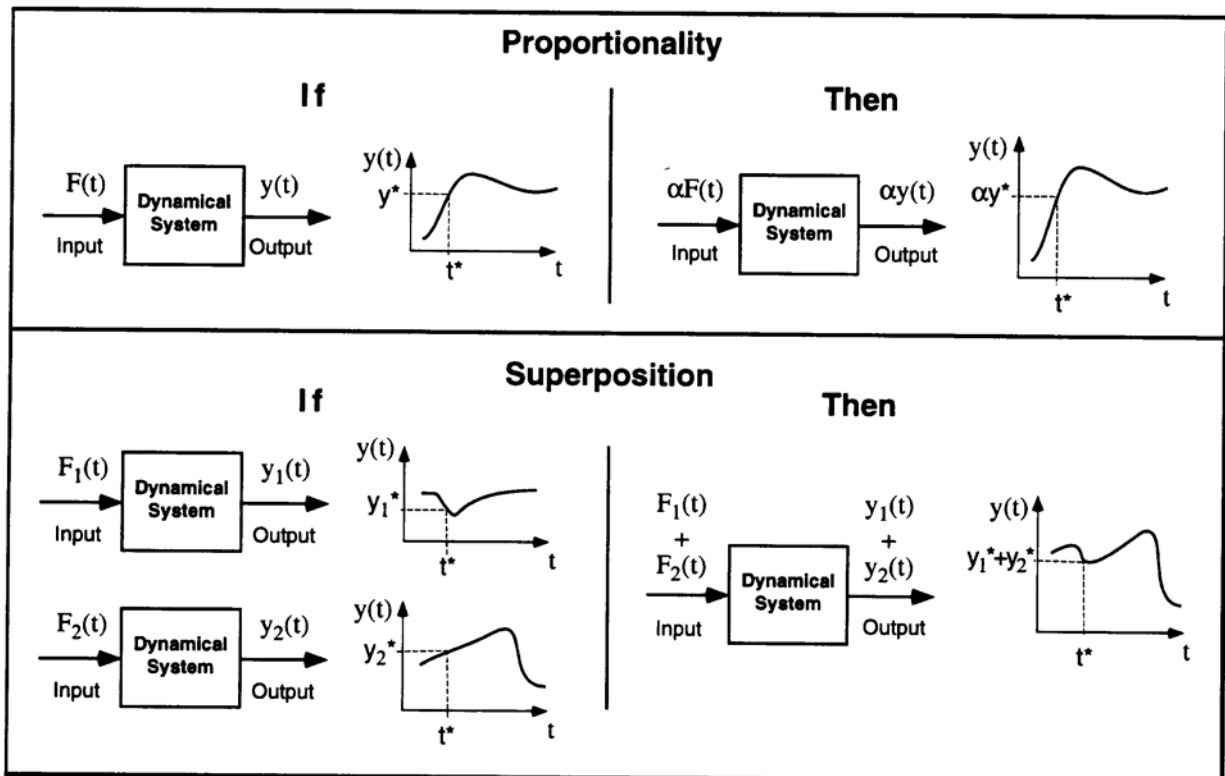


Figure 3.2 The principles of proportionality and superposition.

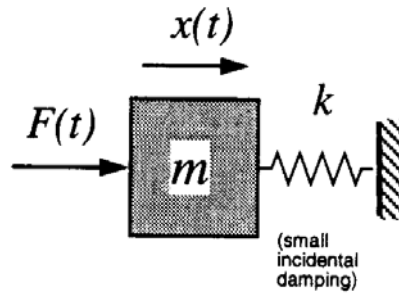


Figure 3.3 System for tests of linearity principles.

2. “Enable” the **Force (Torque)** driving function via the **Setup Driving Function** dialog box. Setup a unidirectional **Impulse** input shape of 5 N, 50 ms pulse width, 1 repetition, and 1000 ms dwell time. Setup to acquire Encoder 1 Position and Drive Input data once every 1 servo cycles.
3. Execute the impulse maneuver (select **Extended Data Sampling** in the **Execute** box) and plot the resulting Encoder 1 position, and Drive Input data over a 1.0s. interval. To obtain proper scaling, it is best to plot position on the left axis, and drive input on the right axis. Print your plot.
4. Change the impulse amplitude to 10 N and repeat Step 3 above.

**Exercise:**

- A. How does changing the forcing function input amplitude affect the output (neglecting mechanism friction effects)? Explain using the terminology defined in Figure 3.2.

Procedure (Superposition):

5. Change the impulse width to 25 ms (10 N amplitude) and repeat Step 3 above. Plot the data over the initial 1.5 s. Measure the times when the output oscillation phase is at  $\pi$ ,  $3\pi/2$ , and  $2\pi$  of the cosine function (i.e. first negative slope zero crossing, first minimum, and first positive slope zero crossing) and identify them as  $t_1$ ,  $t_2$ , and  $t_3$  respectively. You should “zoom” your plot scaling to obtain best resolution to measure these times.
6. Change the number of impulse repetition to 2 and set the dwell period such that the second pulse is centered about  $t_1$ . (You should properly account for the initial and second pulse durations.) Setup to acquire Encoder 1 Position and Drive Input data once every 1 servo cycle. Execute the double pulse input (select **Extended Data Sampling** in the **Execute** box) and plot the position and input data. Print the plot.
7. Repeat Step 6 with the second pulse centered about  $t_2$ . Repeat again for  $t_3$ .

**Exercise:**

- B. Explain the system output for each of the three cases in terms of the principle of superposition using the terminology defined in Figure 2. You may consider  $F_1(t)$  to be the initial impulse, and  $F_2(t)$  to be the second. Select data points at three distinct times from one of the three double pulse cases (your choice. Note that using data points fairly early in the response usually results in measurements that are more consistent with theory because friction effects are less dominant.). At least two of these times should be greater than the second pulse delay time,  $t_i$  ( $i = 1, 2, \text{ or } 3$ ) for whichever case you choose. Show based on measurements of the initial single pulse response (Step 5) how the values for your double pulse case are predicted by with superposition.