Model Reaching Adaptive Control for Vibration Isolation

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Abstract—Adaptive control has drawn attention for active vibration isolation and vehicle suspensions because of its potential to perform in the presence of nonlinearities and unknown or time-varying parameters. Model-reference adaptive control has been used to force the plant to track the states or certain outputs of the ideal reference model. In this brief, we study a new adaptive approach, “model-reaching” adaptive control, to achieve the ideal multi-degree-of-freedom (DOF) isolation effect of a skyhook target without using a reference model. We define a dynamic manifold for the target dynamics in terms of the states of the plant, rather than the error of the plant tracking of the reference. Then we describe an adaptive control law based on Lyapunov analysis to make the isolation system reach the dynamic manifold while estimating the unknown parameters. The proposed method directly employs measurement of the payload velocity and its displacement relative to ground, and the effects of imperfect velocity measurements using a geophone are quantified. We carry out a detailed experimental investigation based on a realistic single degree-of-freedom (SDOF) plant with friction, demonstrate the effectiveness of the proposed adaptive control, and show that the target dynamics of the skyhook isolator are attained. A framework for achieving general targets is also suggested.

Index Terms—Adaptive control, model reaching, skyhook damping, sliding control, vehicle suspension, vibration isolation.

I. INTRODUCTION

ACTIVE vibration isolation systems or suspensions have become necessary in many applications to compensate for the low-frequency inadequacy of passive vibration isolation. A variety of control techniques, such as proportional-integral-derivative (PID) or lead-lag compensation, linear quadratic Gaussian (LQG)/H₂, H∞, μ-synthesis, and feedforward control, have been used in active systems [1]–[9].

One of the classical concepts in the literature on vibration isolation is the “skyhook” damper proposed by Karnopp in 1974 [10], [1]. The skyhook damper is a virtual configuration where the damper is connected with a virtual inertial “sky.” Fig. 1(a) shows a single degree-of-freedom (SDOF) skyhook isolator. In passive systems, the damper can be connected only to the base since there is no practical inertial sky, as shown in Fig. 1(b). The vibration transmissions of the two configurations are compared in Fig. 2, from which we see that whereas there exists a tradeoff between high- and low-frequency performances in Fig. 1(b), there is no such conflict in the skyhook system. The skyhook configuration also eliminates the tradeoff between rejection of disturbances directly acting at the payload and isolation from ground vibration.

Because of these advantages, the skyhook configuration has been a target in many isolation or suspension systems. Sliding control has been used to attain the desired skyhook effect in the presence of uncertainties [11]–[13]. Adaptive control has attracted a great deal of attention because it does not require prior knowledge of the plant parameters and works well in systems with nonlinearities and time-varying parameters. Sunwoo et al. [14] used model-reference adaptive control for vehicle suspensions by tracking the states of the desired skyhook model. Alleyne and Hedrick [15] considered the nonlinear dynamics of an electrohydraulic actuator and developed an adaptive control for tracking the ideal skyhook force of a suspension. Wang and Sinha [16] proposed a model-reference adaptive algorithm to achieve multi-degree-of-freedom (DOF) skyhook isolation by tracking all of the states. Bakhtiarī-Nejad and Karami-Mohammadi [17] considered the flexible mode of a vehicle body and used adaptive control to track the states of a reference model.
of an LQ-controlled skyhook system. Zhang and Alleyné [18]
proposed a position-tracking schedule with adaptive control
to overcome the limitations of an electrohydraulic actuator on
force tacking.

These previous studies share a common point: they use an
adaptive algorithm to track (or follow) the states or certain out-
puts of the desired isolation model. This model-reference adap-
tive control generally requires a measurement (or an observer)
of ground disturbance (velocity or acceleration in inertial frame)
as an input to the reference model, increasing the cost and com-
plexity of such systems, and even making the implementation
impractical in some cases. For example, it is difficult to sense
the road surface while a vehicle is moving. (In the literature,
a sensor is usually mounted at the wheel hub, but the measure-
ment is valid only below approximately 10 Hz.)

In this brief, we experimentally study a novel adaptive control
scheme for vibration isolation without employing model-refer-
ence tracking [13]. The idea is to design a dynamic manifold in
terms of the states of the plant that corresponds to the isolation
target then to use adaptive control to drive the system onto this
manifold while updating the system parameters. This adaptive
algorithm is formulated directly in terms of the readily mea-
sured payload displacement relative to ground and its velocity,
rather than the ground motion. We show that if the velocity is
measured using a geophone, then its corner frequency must be
lower than about half of the corner frequency of the skyhook
target. We carry out an experimental study based on a realistic
SDOF plant with friction and demonstrate the effectiveness of
the proposed adaptive control for vibration isolation. The con-
vergence of parameter estimates is also discussed.

II. ADAPTIVE CONTROL FOR VIBRATION ISOLATION
WITHOUT MODEL REFERENCE

A. Isolation Plant and Target Dynamics

Suppose an \( n \)-DOF isolated platform is subject to excitation
from the ground or base. The governing equation takes the form

\[
M\dddot{x} + C\dot{x} + K(x - x_0) + F_{rc} = Bu
\]  

(1)

where \( M, C, \) and \( K \) are mass, damping, and stiffness matrices
dimension \( n \times n; F_{rc} \) is the friction force matrix; \( B \) is a
\( n \times r \) (\( r \geq n \)) matrix determined from actuator placement
with full-row rank; \( x \) is the vector of displacements; \( x_0 \) is the vector
of ground disturbances; and \( u \) is the control force vector. Al-
though many elaborate friction models to account for static, dy-
namic, and Stribleck frictions are available, in the present study
we take the force \( F_{rc} \) to obey the Coulomb friction model

\[
F_{rc} = F_{\text{sgn}}(\dot{x} - \dot{x}_0)
\]

where \( \text{sgn} \) denotes the signum function. This neither alters the
method nor limits the validity of the results. The parameters of
the \( M, C, K, \) and \( F \) matrices are generally unknown. The ma-
trix \( B \) is determined by the geometric location of the actuators
and sensors, which is relatively easy to obtain.

The ideal “skyhook” system is selected as the target. The
target dynamics of an \( n \)-th order skyhook isolator take the form

\[
\dddot{\hat{x}} + \hat{C}\dot{\hat{x}} + \hat{K}(x - x_0) = 0
\]

(2)

Because the mass matrix \( \hat{M} \) is positive–definite, we can simplify
the skyhook target by normalizing \( \hat{M} \) as identity into the form

\[
\dddot{x} + \hat{C}\dot{x} + \hat{K}(x - x_0) = 0
\]

(2)

where \( \hat{C} \) and \( \hat{K} \) are often block-diagonal matrices, which sug-
gests that we achieve the skyhook isolation for each of the vari-
able \( x_i, i = 1, 2, \ldots n \).

B. Model-Reaching Adaptive Control of Isolation

As mentioned in the Introduction, the conventional way to
achieve the skyhook effect using an adaptive algorithm is to
control the plant to follow the states or output of the target and
use the tracking errors for parameter adaptation. In this section,
we describe a new adaptive control algorithm, which we call
model-reaching adaptive control [13].

Define a dynamic manifold vector in the state–space

\[
\sigma = \dot{x} + (sI + \hat{C})^{-1}\hat{K}(x - x_0)
\]

(3)

where \( s \) is the Laplace operator. Then on the manifold \( \sigma = 0, \)
we have

\[
\dot{x} + (sI + \hat{C})^{-1}\hat{K}(x - x_0) = 0
\]

(4)

which is exactly the target skyhook isolation

\[
\dddot{x} + \hat{C}\dot{x} + \hat{K}(x - x_0) = 0
\]

(2)

In the following, we will describe a method by which adaptive
feedback control can drive the dynamics of the plant to reach the
manifold \( \sigma = 0 \) when the parameters of \( K, C, M, \) and \( F \) are
not known.

Let us first rearrange the unknown parameters in the matrices
\( K, C, M, \) and \( F \) into a column vector \( a \) and denote

\[
K(x - x_0) + C(\dot{x} - \dot{x}_0) - M(sI + \hat{C})^{-1}\hat{K}(x - x_0)
+ F_{\text{sgn}}(\dot{x} - \dot{x}_0) := Ya
\]

(5)

where \( Y \) is a matrix with proper dimension composed of \( x - x_0 \)
and \( \dot{x} - \dot{x}_0 \), which can be measured. (In practice, the relative
velocity \( \dot{x} - \dot{x}_0 \) can also be estimated from \( x - x_0 \).) Note that
in (5) the unknown matrices \( K, C, M, \) and \( F \) show up linearly.

Next, using Lyapunov analysis and Barbalat’s lemma, we de-
rive the control and adaptation laws using a procedure similar to
that in [19]. We choose a positive–define Lyapunov function as

\[
V(\sigma, \tilde{a}) = \frac{1}{2}\sigma(t)^T M\sigma(t) + \frac{1}{2}\tilde{a}(t)^T P^{-1}\tilde{a}(t)
\]

(6)

where the vector \( \sigma(t) \) is defined by (3), \( M \) is the (positive–define
) mass matrix of the system, \( P \) is a preselected (constant)
symmetric positive–define matrix, and the vector \( \tilde{a}(t) \) is the
error vector of online estimates of the parameters \( a \). The time
derivative of \( V(\sigma, \tilde{a}) \) is

\[
\dot{V}(\sigma, \tilde{a}) = \sigma(t)^T M\dot{\sigma}(t) + \dot{\tilde{a}}(t)^T P^{-1}\tilde{a}(t).
\]

(7)
Using (1) and (3), we obtain
\[
\dot{V}(\sigma, \dot{\sigma}) = \sigma^T [M\dot{\sigma} + M(sI + C)^{-1}K(x - \dot{x}_0)] + \dot{\sigma}^T P^{-1} \dot{\sigma}(t)
\]
\[
= \sigma^T [(Bu - K(x - \dot{x}_0) - C(x - \dot{x}_0)] - F_s \text{sgn}(x - \dot{x}_0)
\]
\[
+ \dot{M}(sI + C)^{-1}K \dot{s}(x - \dot{x}_0) + \dot{\sigma}^T P^{-1} \dot{\sigma}(t).
\]
Substituting the expression (5) into the previous equation, we obtain
\[
\dot{V}(\sigma, \dot{\sigma}) = \sigma^T (Bu - Y a) + \dot{\sigma}^T P^{-1} \dot{\sigma}(t).
\]
We choose the control-force vector as
\[
u = B^{-1} \dot{Y} - k_d \sigma(t)
\]
where the matrix \(k_d\) is a selected positive–definite matrix of \(n \times n\), the vector \(\dot{\sigma}\) is the online estimate of the unknown parameters of \(\sigma\), and the estimation error \(\dot{\sigma} = \dot{\sigma} - \dot{\sigma}\). Note that \(B \in R^{n \times r}\) with full row rank and pseudoinverse can be used for \(B^{-1}\) in the case of \(r > n\). We substitute (10) into (9) and obtain
\[
\dot{V}(\sigma, \dot{\sigma}) = -\sigma^T k_d \sigma + \sigma^T Y (\dot{\sigma} - a) + \dot{\sigma}^T P^{-1} \dot{\sigma}(t)
\]
\[
= -\sigma^T k_d \sigma + \sigma^T Y (\dot{a} - a) + \dot{\sigma}^T P^{-1} \dot{\sigma}(t).
\]
Hence, if we choose the parameter adaptation law as
\[
\dot{\dot{\sigma}}(t) = \dot{\sigma} = -P Y \sigma(t)
\]
we have
\[
\dot{V}(\sigma, \dot{\sigma}) = -\sigma^T (Bu - Y a).
\]
Then \(\dot{V}(\sigma, \dot{\sigma})\) is negative–semidefinite. We can prove further that \(\dot{V}(\sigma, \dot{\sigma})\) is bounded. Thus, according to the Lyapunov theorems and Barbalat’s lemma [20], we conclude that \(\sigma(t) \to 0\) as \(t \to \infty\). Therefore, using the adaptive control (10) and (12), we drive the states of the system to reach the manifold (3) upon which the plant achieves the target dynamics of skyhook isolation (2). We call this adaptive algorithm model-reaching adaptive control. Note that to implement this adaptive isolation control, we only need to measure \(\dot{x}\) and \(x - \dot{x}_0\).

Furthermore, the manifold (3) is dynamic, in the sense that there is a Laplace operator therein. This provides some flexibility to select the initial state and, therefore, the initial value of \(\sigma\), and thereby change the transient properties as \(\sigma(t) \to 0\). So theoretically we can choose the initial state in \(\sigma\) to ensure transient performance of vibration isolation. The practical implementation of this idea remains a topic of investigation.

The adaptive control works even if payload mass or other parameters change slowly (relatively to the rate of adaptation) or intermittently. The selection of the constant matrices of \(P\) and \(k_d\) can be used to adjust the time of adaptation and the time to reach the manifold. Like model-reference adaptive control, the adaptation law (12) cannot ensure that the parameters converge to their true values unless the system is persistently excited [20]; that is, there exist \(\alpha\) and \(\delta\) such that
\[
\int_{t_0}^{t_0 + \delta} Y^T Y dt \geq \alpha I, \quad \forall t_0 \geq 0.
\]

C. Effect of Geophone Dynamics

In the foregoing derivation of the adaptive controller, we assume that the absolute velocity of the isolated platform can be measured. But in practice, velocity measurements are only valid above a certain frequency. For a geophone sensor the measured output \(\dot{x}_d\) and the actual velocity \(\dot{x}_d\) generally take the form
\[
\dot{x}_d = \frac{s^2}{s^2 + 2\omega_y s + \omega_y^2} \dot{x} + \frac{2\omega_y}{s^2 + 2\omega_y s + \omega_y^2} \dot{x}_0.
\]
where \(\omega_y\) and \(\zeta_y\) are the resonance frequency and damping ratio of the geophone sensor. With the measurement \(\dot{x}_d\), the actual dynamic manifold becomes
\[
\sigma = \dot{x}_d + (s + C)^{-1} K(x - \dot{x}_0).
\]
Suppose that the target dynamics for all the DOFs are selected as skyhooks with frequency \(\omega_s\) and damping \(\zeta_s\); that is, \(\mathbf{K} = \text{diag}([\omega_s^2, \omega_s^2, \ldots, \omega_s^2])\) and \(\mathbf{C} = \text{diag}([2\zeta_s \omega_s^2, 2\zeta_s \omega_s^2, \ldots, 2\zeta_s \omega_s^2])\). By plugging (15) into (16) and using the Routh–Hurwitz criterion, we conclude that the dynamics of \(x\) on the actual dynamic manifold \(\sigma = 0\) are stable if
\[
\frac{\omega_s}{\omega_y} > \frac{\zeta_s + \zeta_y}{\zeta_s - \zeta_y}.
\]
This suggests that the geophone resonance frequency should be smaller than half of the resonance frequency of the target skyhook isolator.

III. EXPERIMENT AND RESULTS

In order to verify the control strategy and to demonstrate the effectiveness of the proposed adaptive control, we carry out an experimental investigation. An electromagnetic shaker is
adapted so that the armature (mounted via flexures to the stator) and a mass block fixed on it compose a SDOF isolated platform and the voice coil serves as actuator, as seen in Fig. 3(a). A magnetically-shielded geophone is mounted onto the platform to measure its absolute velocity \( \dot{x} \), and an eddy–current gap sensor is used to measure the relative displacement \( x - x_0 \). The isolation system is set on a wood benchtop. Because the mass of the platform is far less than that of the base (stator and bench), we can ignore the effect of the control force \( u \) on base vibration. A second geophone is set on the base to monitor its vibration, but is not used in control. The sensor signals are connected to 16-b analog-to-digital converters (ADCs) after gain adjustment. Low-pass filters (at 3 kHz) are used to reduce high-frequency noise and aliasing. A 14-b digital-to-analog converter (DAC) and a voltage-to-current power amplifier are used for actuation. The control is implemented using a dSpace 1103 board hosted by a PC. We set the sampling frequency to 10 kHz. The whole system is shown in Fig. 3(b).

This is a single-DOF isolation platform. If we take the control signal \( u \) as voltage, \( B \) is a scalar with units of N/V. We normalize \( B \) to one and write the plant model as

\[
mx\ddot{x} + c(x - x_0) + k(x - x_0) + f \text{sgn}(\dot{x} - \dot{x}_0) = u
\]

where \( k, c, m, \) and \( f \) are unknown. Note that due to the normalization of \( B \), the units of \( k, c, m, \) and \( f \) are now N/m/(N/V), N-s/m/(N/V), kg/(N/V), and N/(N/V), respectively. According to the parameterization (5) in Section II, we write

\[
a = [k, c, m, f]^T
\]

\[
Y = \left[ x - x_0, \dot{x} - \dot{x}_0, \frac{k_s}{s + c}(x - x_0), \text{sgn}(\dot{x} - \dot{x}_0) \right]^T
\]

where \( \dot{x} - \dot{x}_0 \) is estimated by passing \( x - x_0 \) through a filter \( s/(1 + \tau s) \) with a pole at 1.5 kHz. The passive isolation system (open loop) has a natural frequency of around 12 Hz, and we set our target as a skyhook isolator with a natural frequency of 1.2 Hz and damping ratio of 0.7. To satisfy the condition (17) we correct our geophone corner frequency from 5 to 0.5 Hz and damping to 0.7 using a second-order circuit [21]. In the following results, we select the constant \( k_d \) as 3000 and the constant matrix \( P \) as \( \text{diag}([1e14, 1e10, 3e8, 1e3]) \). Note that the value of \( P_{ii}, i = 1, \ldots, 4 \), can be adjusted through several trials so that the parameters of \( \dot{k}, \dot{c}, \dot{m}, \) and \( \dot{f} \) can be adapted at similar rate, since here \( \dot{a}_i(t) = -\int_0^t P_{ii}Y_i(t)\sigma(t) \, dt \).
Fig. 8. Convergence of parameter estimations under 10-Hz base excitation: stiffness $\hat{k}$ in N/m/(N/V), damping $\hat{c}$ in N s/m/(N/V), mass $\hat{m}$ in kg/(N/V), and friction $\hat{f}$ in N/(N/V).

Fig. 9. Time response of isolated platform when adaptive control turns on under random excitation. Base velocity (dot), platform velocity (solid), target skyhook velocity (dash).

We employ a second shaker as a reaction-mass actuator to excite the base. Fig. 4 shows the time responses when the adaptive control is turned on while the base is excited at 10 Hz by the second shaker in addition to ambient excitation. The initial guesses of the parameters $k, c, m$, and $f$ are selected as zeros. In this figure, we show the measured velocity $\hat{x}$ of the platform, measured velocity $\hat{x}_0$ of base vibration, and the calculated velocity $\hat{x}_g$ of target skyhook isolator. We see that the vibration of the passive isolated platform (control off) is amplified, since the base vibration is close to the resonant frequency of 12 Hz. After the control turns on, the platform isolation tends to the target skyhook output in a few seconds. Fig. 6 shows the zoomed time response of the controlled isolator. We see that the proposed adaptive algorithm can effectively control the platform to match the target skyhook isolation.

In the zoomed velocity plots there are pulses when the velocity crosses zero; this is because the Coulomb friction model is not valid at zero velocity. The other small residual errors are due to sensor noise and some unmodeled dynamics (such as the 3 kHz-low-pass filters for the sensors). For comparison, we also implement the model-reaching adaptive control by ignoring the friction term in the model. Fig. 5 shows the zoomed time response of such a controlled isolator with the same $P$ and $k_d$. Comparing Figs. 5 and 6, we see that although the Coulomb friction model is not valid at zero velocity, we obtain a performance improvement by taking it into account. The effort of the friction compensation can also been seen in the control force as sudden changes in voltage shown in Fig. 7.

The parameter convergence for 10-Hz base excitation is shown in Fig. 8. The damping and friction converge to reasonable values close to their offline estimates. But the estimated mass is negative. This can be explained by using the condition (14) to check for persistent excitation by starting at any time $t_0 \geq 0$ and checking the integral over any time interval $\delta$. One typical
value of the integral \( \int_{t_0}^{t_0+\delta} Y^T Y \, dt \) over 2 s for 10-Hz base excitation is

\[
\begin{bmatrix}
2.638e-11 & 3.512e-11 & -1.455e-9 & 2.483e-8 \\
3.512e-11 & 3.783e-7 & -3.095e-8 & 7.759e-4 \\
-1.455e-9 & -3.095e-8 & 8.275e-8 & -6.156e-5 \\
2.483e-8 & 7.759e-4 & -6.156e-5 & 2.0000 \\
\end{bmatrix}
\]

whose singular values are 2.0, 8.64e-8, 7.18e-8, and 0. The integrals of other time intervals are similar. This indicates that the system is not persistently excited. Examining the expression for \( Y \) given by (19), we can understand this result more intuitively. With our choice of skyhook target with a corner frequency of 1.2 Hz and a damping ratio of 0.7, \((\tilde{K}s)/(s+\tilde{\gamma})\) is a high-pass filter at 1.58 Hz. Thus, for 10-Hz excitation, the third element \(-\tilde{K}I(s)/\tilde{K}(x-x_0)\) of \( Y \) closely approximates \(-\tilde{K}(x-x_0)\), which is proportional to the first element \( x-x_0 \) of \( Y \). Hence, the mass adaptation error can be approximately cancelled by a contribution from the stiffness and, thus, their adaptation need not converge the actual values. Therefore, although the desired skyhook isolation is achieved under nonpersistent excitation, not every physical parameter can be uniquely identified. Only the parameters (or combination of parameters) which govern the system’s behavior under the excitation can be identified.

Fig. 9 shows the time responses of the isolated platform when the adaptive control is turned on while the base is subject to random excitation by the shaker plus the ambient disturbance. (The actual spectrum of the base vibration is not white, because of the bench dynamics and the bandwidth limitation of reaction-mass excitation by the second shaker.) The initial parameters are selected as zero. We see that the desired isolation effect of the skyhook target is reached very quickly.

To examine the effect of the matrix \( P \), we reduce \( P \) by a factor of 10, from \( \text{diag}([1e14, 1e10, 3e8, 1e3]) \) to \( \text{diag}([1e13, 1e9, 3e7, 1e2]) \). Figs. 10 and 11, respectively, show the time response and parameter estimates when the adaptive control turns on under 10-Hz base isolation. Comparing these two figures with Figs. 4 and 8, we see that the transient time has become longer due to smaller \( P \). The final values of the parameter adaptation are similar to those obtained before.

IV. CONCLUDING REMARKS

In this brief, we study a new adaptive algorithm to achieve target dynamics (skyhook isolation) without model reference. This algorithm employs directly measurements of payload absolute velocity and relative displacement, and has the potential to improve transient performance. Its derivation is based on Lyapunov analysis and Barbalat’s lemma. The main idea here is to design a dynamic manifold for the target, rather than control the plant to follow the model reference, so it can be taken as an extension of model-reaching sliding control [13], [22], [23] and adaptive sliding control [19].

The control and adaptation laws are derived for general single- or multi-DOF isolation systems, and the effects of geophone dynamics are also examined. We further carry out an experimental investigation based on a realistic plant with friction. The experiments indicate that this control strategy is highly effective for active vibration isolation without prior knowledge of system parameters. We also note that the choice of the matrices \( P \) and \( k_d \) is very important for the transient time. The strategy of performing real-time updates of the matrix \( P \) [24] might be used, or some slow updating schedule (to save computation) might be further explored.

In this brief, a dynamic manifold is designed for skyhook target dynamics. But our approach can be easily extended to more general desired targets by modification of (3) so that

\[
\sigma = \dot{x} + L(s)(x-x_0)
\]
where $L(s)$ is a linear operator. We can design $L(s)$ for different performance requirements or different disturbances, for example, to account for the spectrum of ground vibration. It is interesting to note that the choice of $L(s)$ can be recast as a feedback problem as shown in Fig. 12. The corresponding model-reaching adaptive control can be obtained thereafter.

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